

LIMITI

A) Di funzioni elementari

1. $\lim_{x \rightarrow +\infty} x^a = +\infty$	se $a > 0$	6. $\lim_{x \rightarrow +\infty} \log_a x = +\infty$	se $a > 1$
2. $\lim_{x \rightarrow +\infty} x^a = 0$	se $a < 0$	7. $\lim_{x \rightarrow +\infty} \log_a x = -\infty$	se $0 < a < 1$
3. $\lim_{x \rightarrow +\infty} \sqrt[n]{x} = +\infty$	essendo $\sqrt[n]{x} = x^{\frac{1}{n}}, n \in \mathbb{N}$	8. $\lim_{x \rightarrow 0^+} \log_a x = -\infty$	se $a > 1$
4. $\lim_{x \rightarrow 0^+} x^a = +\infty$	se $a < 0$	9. $\lim_{x \rightarrow 0^+} \log_a x = +\infty$	se $0 < a < 1$
5. $\lim_{x \rightarrow 0^+} x^a = 0$	se $a > 0$		

B) Notevoli

10. $\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$	23. $\lim_{x \rightarrow 0} \frac{(\sin x)^n}{x^n} = 1$
11. $\lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^x = e^a$	24. $\lim_{x \rightarrow 0} \frac{\sin kx}{kx} = 1, k \neq 0$
12. $\lim_{x \rightarrow 0} \left(1 + \frac{1}{x}\right)^{\frac{1}{x}} = e$	25. $\lim_{x \rightarrow 0} \frac{\sin kx}{x} = k, k \neq 0$
13. $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$	26. $\lim_{x \rightarrow 0} \frac{\sin kx}{hx} = \frac{k}{h}, k, h \neq 0$
14. $\lim_{x \rightarrow 0} \frac{\ln_a(1+x)}{x} = \log_a e = \frac{1}{\ln a}, a \neq 1$	27. $\lim_{x \rightarrow 0} \frac{\sin kx}{\sin hx} = \frac{k}{h}, k, h \neq 0$
15. $\lim_{x \rightarrow 0} \frac{\ln(1+ax)}{x} = a$	28. $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$
16. $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$	29. $\lim_{x \rightarrow 0} \frac{\arctan x}{x} = 1$
17. $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a = \frac{1}{\log_a e}$	30. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$
18. $\lim_{x \rightarrow 0} \frac{(1+x)^k - 1}{x} = k; k \in \mathbb{R}$	31. $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$
19. $\lim_{x \rightarrow +\infty} \frac{e^x}{x^\beta} = +\infty$	32. $\lim_{x \rightarrow 0} \frac{1 - (\cos x)^2}{x^2} = 1$
20. $\lim_{x \rightarrow +\infty} \frac{(\ln x)^\alpha}{x^\beta} = 0$	33. $\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0; (\text{si ha } x \sin \frac{1}{x} < x)$
21. $\lim_{x \rightarrow +\infty} x^\beta \cdot (\ln x)^\alpha = 0; \forall \alpha, \beta \in \mathbb{R}, \beta > 0$	34. $\lim_{x \rightarrow 0} x^2 \left(\sin \frac{\pi}{x}\right)^2 = 0$ (poichè $0 \leq x^2 \left(\sin \frac{\pi}{x}\right)^2 \leq x^2$)
22. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$	

Osservazioni:

- $\lim_{x \rightarrow \pm\infty} \sin x; \lim_{x \rightarrow \pm\infty} \cos x; \lim_{x \rightarrow \pm\infty} \tan x$ **non esistono**;
- $\log_a x < x^a < a^x$