LIMITI

A) Di funzioni elementari

1. $\lim_{x \to +\infty} x^a = +\infty$ se a>0	6. $\lim_{x \to +\infty} \log_a x = +\infty$ se a>1
2. $\lim_{x \to +\infty} x^a = 0$ se a<0	7. $\lim_{x \to +\infty} \log_a x = -\infty$ se 0<a<1
3. $\lim_{x \to +\infty} \sqrt[n]{x} = +\infty$ essendo $\sqrt[n]{x} = x^{\frac{1}{n}}, n \in \mathbb{N}$	8. $\lim_{x \to 0^+} \log_a x = -\infty$ se a>1
4. $\lim_{x \to 0^+} x^a = +\infty$ se a<0	9. $\lim_{x \to 0^+} \log_a x = +\infty$ se 0<a<1
5. $\lim_{x \to 0^+} x^a = 0$ se a>0	

B) Notevoli

10. $\lim_{x \to \infty} (1 + \frac{1}{x})^x = e;$

11. $\lim_{x \to 0} (1 + x)^{\frac{1}{x}} = e$

12. $\lim_{x \to \infty} (1 + \frac{a}{x})^x = e^a$

13. $\lim_{x \to 0} (1 + \frac{x}{a})^{\frac{1}{x}} = e^{\frac{1}{a}}$

14. $\lim_{x \to \infty} (1 + x)^{\frac{1}{x}} = 1$

15. $\lim_{x \to 0} (1 + \frac{1}{x})^x = 1$

in generale si hanno: da 16 a 21

16. $\lim_{x \to \infty} \left(1 + \frac{1}{f(x)}\right)^{f(x)} = e$

17. $\lim_{x \to 0} (1 + f(x))^{\frac{1}{f(x)}} = e$

18. $\lim_{x \to \infty} \left(1 + \frac{a}{f(x)}\right)^{f(x)} = e^a$

19. $\lim_{x \to 0} \left(1 + \frac{f(x)}{a}\right)^{\frac{1}{f(x)}} = e^{\frac{1}{a}}$

20. $\lim_{x \to 0} \left(1 + \frac{1}{f(x)}\right)^{f(x)} = 1$

21. $\lim_{x \to \infty} (1 + f(x))^{\frac{1}{f(x)}} = 1$

22. $\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$

23. $\lim_{x \to 0} \frac{\ln_a(1+x)}{x} = \log_a e = \frac{1}{\ln a}, a \neq 1$

24. $\lim_{x \to 0} \frac{\ln(1+ax)}{x} = a$

25. $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

26. $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a = \frac{1}{\log_a e}$

27. $\lim_{x \to 0} \frac{(1+x)^k - 1}{x} = k; \ k \in \mathbb{R}$

28. $\lim_{x \to +\infty} \frac{e^x}{x^\beta} = +\infty$

29. $\lim_{x \to +\infty} \frac{(\ln x)^\alpha}{x^\beta} = 0$

30. $\lim_{x \to +\infty} x^\beta \cdot (\ln x)^\alpha = 0; \ \forall \alpha, \beta \in \mathbb{R}, \beta > 0$

31. $\lim_{x \to 0} \frac{\sin x}{x} = 1$

32. $\lim_{x \to 0} \frac{(\sin x)^n}{x^n} = 1$

33. $\lim_{x \to 0} \frac{\sin kx}{kx} = 1, \ k \neq 0$

34. $\lim_{x \to 0} \frac{\sin kx}{x} = k, \ k \neq 0$

35. $\lim_{x \to 0} \frac{\sin kx}{hx} = \frac{k}{h}, \ k, h \neq 0$

36. $\lim_{x \to 0} \frac{\sin kx}{\sin hx} = \frac{k}{h}, \ k, h \neq 0$

37. $\lim_{x \to 0} \frac{\tan x}{x} = 1$

38. $\lim_{x \to 0} \frac{\arctan x}{x} = 1$

39. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

40. $\lim_{x \to 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$

41. $\lim_{x \to 0} \frac{1 - (\cos x)^2}{x^2} = 1$

42. $\lim_{x \to 0} x \sin \frac{1}{x} = 0; (si\ ha \ |x \sin \frac{1}{x}| < |x|)$

43. $\lim_{x \to 0} x^2 \left(\sin \frac{\pi}{x}\right)^2 = 0$

$(poichè\ 0 \leq x^2 \left(\sin \frac{\pi}{x}\right)^2 \leq x^2)$

Osservazioni:

1. $\lim_{x \to \pm\infty} \sin x \ ; \ \lim_{x \to \pm\infty} \cos x \ ; \ \lim_{x \to \pm\infty} \tan x$
non esistono ;

2. $\log_a x < x^a < a^x$